

Characteristic Subspace:

The non-null vector space formed by the eigen vectors of a matrix A corresponding to an eigen value λ , together with the null-vector, is said to be the characteristic subspace corresponding to λ .

Algebraic multiplicity and Geometric multiplicity

For an n -fold eigen value λ , n is called the algebraic multiplicity of λ .

The rank of the characteristic subspace corresponding to λ is called the geometric multiplicity of λ .

Since the characteristic subspace is always a non-null subspace, it follows that for an eigen value λ ,

$$1 \leq \text{geometric multiplicity} \leq \text{algebraic multiplicity}$$

Definition.

An eigen value λ is said to be regular if the geometric multiplicity of λ is equal to its algebraic multiplicity.

Assignment

Q Find the algebraic and geometric multiplicities of each eigen value of the following matrices

$$(i) \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(ii) \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$