Utility: Cardinal Approach

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★ Cardinal Utility

In the cardinal framework, total utility obtained from a commodity refers to the amount of satisfaction that a consumer obtains from consuming it. Upto a certain point, the more units of a commodity is consumed per unit of time, the more units of satisfaction is received from it. Contributions along this line came from Jeremy Bentham, Leon Walras, Alfred Marshall and others.

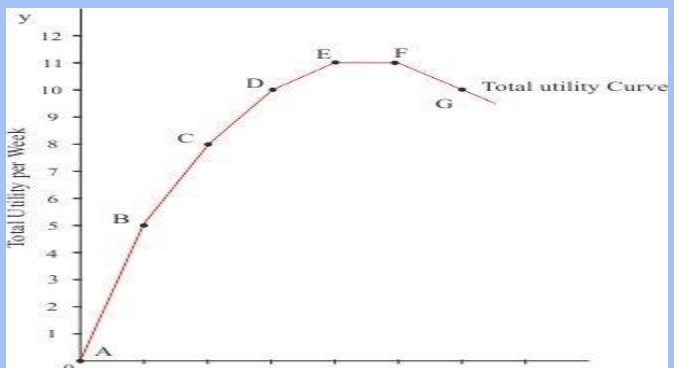
★ Utility and Choice

For understanding how a household will choose from different choices, we have to look at what they can afford, i.e., on budget constraints and the satisfaction they are obtaining from consumption. They have to face a tradeoff between these two.

★ Total Utility (TU)

It refers to the total amount of satisfaction received by a consumer by consuming a commodity. Upto a certain point of time TU increases with increasing levels of consumption. Then it reaches a maximum. After this point, it does not yield higher satisfaction even if more is consumed. This point is called the saturation point for the commodity. Upto this saturation point, the commodity is 'good' to the consumer, beyond that point it is 'bad'.

The diagram shows the Total Utility curve. Saturation point is at E, after



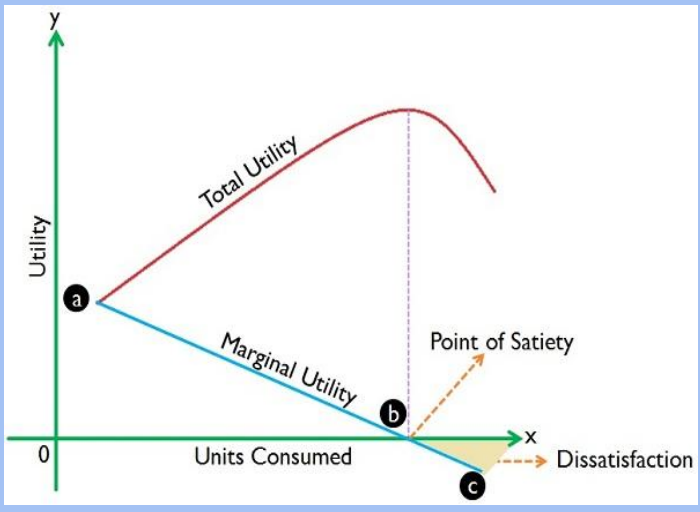
which utility starts to fall with increasing consumption of the commodity.

★ Marginal Utility (MU)

Marginal utility (MU) is the change in utility resulting from one unit increase in consumption of the commodity. Following table shows an example.

Units of commodity consumed	TU	MU
0	0	0
1	9	9
2	16	7
3	21	5
4	24	3
5	25	1

The table shows that the TU when the consumer does not consume anything is zero and it is nine when the consumer buys the first unit. Now, since MU is the change in total utility associated with consumption of one extra unit, and since the first unit adds nine to TU, we can say that MU of the first unit is nine. Similarly MU of the second, third, fourth and fifth units are seven, five, three and one respectively.



MU curve can be drawn from the TU curve as shown in the diagram beside. When TU reaches at the saturation point, MU falls to zero. Thereafter, MU becomes negative, as additional units of the commodity increases dissatisfaction of the consumers.

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★ Diminishing Marginal Utility (DMU)

The law of diminishing marginal utility states that as more and more of a commodity is consumed, its marginal utility diminishes. For example, the consumption of a television-its marginal utility falls after the second or third hour, and will be negative after the fourth or fifth hour.

★ Marshallian Theory of Marginal Utility

The Marshallian utility theory is based on the following hypotheses:

i) **Cardinal Measurability:** Utility obtained by the consumer from the consumption of a commodity is indirectly measurable in terms of money. The utilities derived from different commodities are cardinally comparable and amenable to basic arithmetic operations.

ii) **Independence:** The utility which the consumer derives from each commodity purchased is function of that commodity alone:

U = U(X) = U (x1, x2, ...xn) = U1(X1) + U2(X2) +.....+ Un(Xn)

that is, total utility derived from a bundle of commodities is the sum of utilities obtained from the consumption of each commodity separately.

iii) **Diminishing Marginal Utility:** Marginal utility diminishes continuously, i. e. $\frac{\partial U}{\partial x_i}$ and $\frac{\partial^2 U}{\partial x_i^2} < 0$, (i = 1, 2, ...n) , after a certain stage of consumption.

iv) Marginal utility of money is constant. Symbolically, $\frac{\partial U}{\partial M} = \lambda$, where M = Money Income and λ= Marginal utility of money.

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The consumer, who has a fixed amount of money M to spend in a market in which the prices of all commodities are given, will come to equilibrium at a position in which marginal utility of each commodity is proportional to its price.

Formal Proof

Let the price vector be P = (p1, p2, ...pn)

Vector representing consumption bundle X = (x1, x2, ...xn)

M=the amount of money allotted for buying consumption goods

Utility Function is: U(X) = U(x1, x2, ...xn)= U1(x1) + U2(x2) + ...+ Un(xn)

Now, marginal utility of the ith commodity = $\frac{\partial U}{\partial x_1} = \frac{\delta U1}{\delta x1}$

We assume that prices are given, objective of the consumer is to maximize satisfaction (utility) and the consumer spends the entire amount (M) allotted for consumption.

Formally,

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Max U = U(X), subject to M (given) = PX = $p_1x_1 + p_2x_2 + \ldots\ldots\ldots + p_nx_n$
Let us form the Lagrangian function:

$L = U(x_1, x_2, \ldots\ldots, x_n) + \lambda(M - p_1x_1 - p_2x_2 - \ldots\ldots\ldots - p_nx_n)$
Where λ (>0) is a Lagrange multiplier.

The first order conditions for maximization are:

$$\frac{\delta L}{\delta x_1} = \frac{\delta U}{\delta x_1} - \lambda p_1 = 0$$

$$\frac{\delta L}{\delta x_2} = \frac{\delta U}{\delta x_2} - \lambda p_2 = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\delta L}{\delta x_n} = \frac{\delta U}{\delta x_n} - \lambda p_n = 0$$

$$\frac{\delta L}{\delta \lambda} = M - p_1x_1 - p_2x_2 \ldots\ldots\ldots\ldots - p_nx_n = 0$$

From the above equations we get

$$\frac{\frac{\delta U}{\delta x_1}}{p_1} = \frac{\frac{\delta U}{\delta x_2}}{p_2} = \cdots\ldots\ldots\ldots = \frac{\frac{\delta U}{\delta x_n}}{p_n} = \lambda$$

i.e., marginal utility per unit of money must be equal for all the commodities. In equilibrium of the consumer, amount of additional utility derived from an additional Rupee spent, i.e. Marginal utility of money, is λ and equal for all the commodities. The second order condition for equilibrium requires that marginal utility for each commodity must be diminishing. This condition is ensured by assumption.

Criticism

i) The concept of cardinal utility is itself unrealistic as a psychological concept like utility cannot be measured cardinally.

ii) The concept of Independent utility is unrealistic. Marshall assumed that utility derived from a commodity depends on its own consumption alone. But later economists opined that this is unrealistic, as the utility, derived from the consumption of a commodity, does not depend on the consumption of the commodity alone but also on the consumption of related goods (substitutes and complements). For example, utility of tea depends on the consumption of sugar (complement) and coffee (substitute) also.

iii) The idea of constant marginal utility of money is unrealistic. Contradicting the cardinalist view, the ordinalists opined that, like all other commodities, marginal utility of money, too, diminishes with increased possession of money.

★ Derivation of Demand curve from MU curve

The demand curve can be derived using the law of diminishing marginal utility and the consumer's utility maximizing condition. A consumer's demand curve for a commodity graphically shows the relationship between the price of the commodity and the quantity purchased of that commodity under the assumption that all other factors affecting demand remain constant.

The diagram shows at X1 Marginal Utility is MU1, which is equal to P1 (by definition). Similarly, at X2 price is P2, at X3 price is P3 and so on. Joining these points we will get the demand curve.

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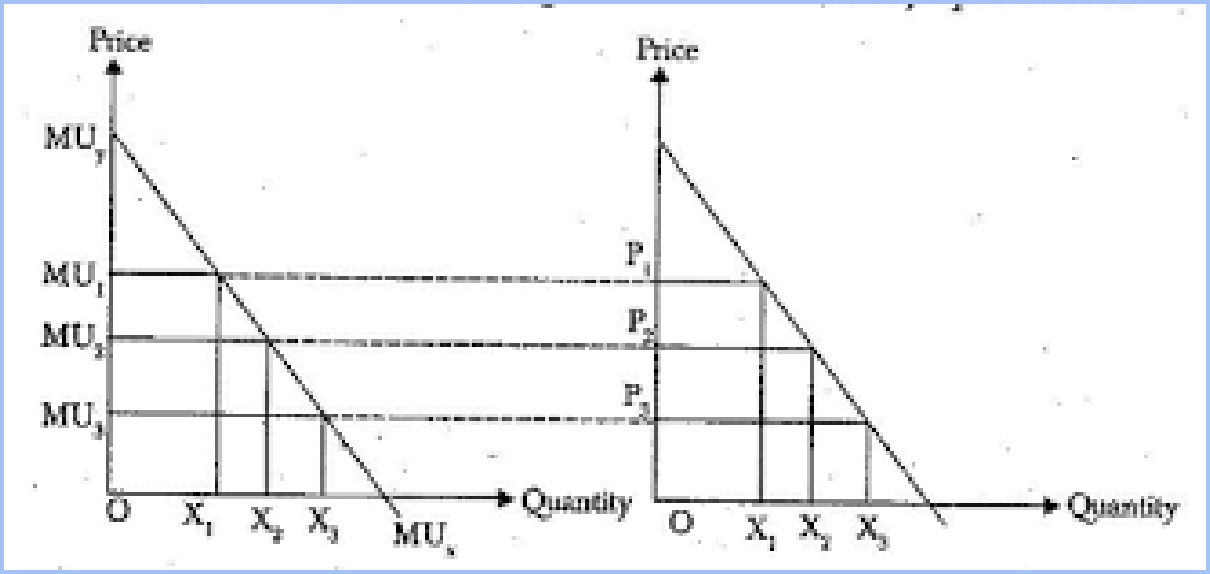

Suppose there are two commodities, x and y. The utility maximization problem is

Max U (x, y),

Subject to $p_x x + p_y y = m$ (where p_x and p_y are the prices of x and y commodities respectively and m is the income of the consumer.

Let $(\bar{x}, \bar{y})$ be the units of commodities x and y that solve $\frac{MU(x)}{MU(y)} = \frac{p_x}{p_y}$ and $p_x x + p_y y = m$.

Now suppose, p_x increases to p_x'. Thus, the ratio $\frac{MU(\bar{x})}{MU(\bar{y})}$ is smaller than $\frac{p_x'}{p_y}$. The law of diminishing marginal utility demands that the consumer must decrease x to say $(\bar{x} + \Delta\bar{x})$ and increase y to $(\bar{y} + \Delta\bar{y})$ to achieve the equality $\frac{MU(\bar{x}+\Delta\bar{x})}{MU(\bar{y}+\Delta\bar{y})} = \frac{p_x'}{p_y}$, where $\Delta\bar{x} < 0$ and $\Delta\bar{y} > 0$

Therefore, $(\bar{x} + \Delta\bar{x}, p_x')$ is another point on the demand curve. Since, $(\bar{x} + \Delta\bar{x}) < \bar{x}$ and $p_x' > p_x$, it is obvious that the quantity demanded falls as the price increases. The process can be repeated for many times and by the resulting points we can get the demand curve for the commodity.