

Ordinal Utility

Compiled by Dr. Gargi Basu of S.A. Jaipuria College

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According to the ordinalists, although we cannot measure utility cardinally, we may compare the utility derived from different commodity bundles in terms of their intensity and arrange them in increasing or decreasing order. If Q_1 , Q_2 and Q_3 be 3 commodity bundles, then utilities derived from these bundles, say $U(Q_1)$, $U(Q_2)$, $U(Q_3)$ may be compared, e.g., $U(Q_1) < U(Q_2) < U(Q_3)$. The above statement reveals only the order of preference but not the magnitude of preference. For example, we may state that the consumer derives more utility from Q_2 than Q_1 , but we cannot say to what extent or how much more utility is derived from Q_2 compared to Q_1 . In this case, we may attach higher numbers to indicate higher levels of utility. But these numbers have no cardinal significance. Suppose we attach the numbers 1 and 2 to two commodity bundles to indicate that the second bundle possesses more utility than the first, but this does not mean that the utility derived from the second bundle is twice of the utility derived from the first bundle. It does not matter if we attach the numbers 7 and 11 with the two bundles respectively. What matters here is the order and relative intensity of utility and not the absolute magnitudes.

★ Properties of Ordinal Utility Function

- i) Utility function is ordinal and unique up to a monotone transformation. Between two commodity bundles Q_1 and Q_2 , either $U(Q_1) > U(Q_2)$ or $U(Q_1) < U(Q_2)$ or $U(Q_1) = U(Q_2)$ and if U is a utility function then $F(U)$, which is any monotone transformation of U is also a utility function.
- ii) The utility function is continuous with continuous first and second order partial derivatives.
- iii) The components of the commodity bundle are perfectly substitutable.

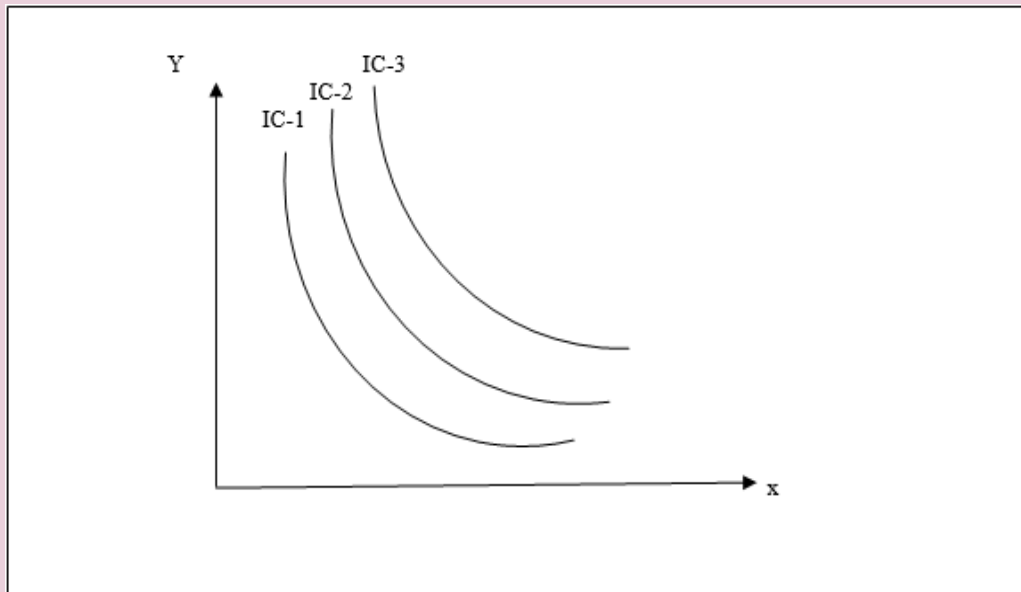
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The geometric counterpart of the utility function, which has wide applications in the Ordinal theory of “Consumer Behaviour” is called Indifference Curve. For two dimensional presentation in diagrams we are to assume that the consumer consumes two commodities only.

★ Indifference Curve

An indifference curve is the locus of points having the same level of utility in a two dimensional commodity space. The positive quadrant containing all the indifference curves is called the “Indifference Map” as depicted below:



In the above indifference map, there are three indifference curves 1, 2 and 3. Here IC-3 indicates higher level of utility than IC-2 and IC-2 than IC-1, i.e $U(IC3) > U(IC2) > U(IC1)$.

★ Properties of Indifference Curve

Indifference curves are drawn on the basis of the following properties:

- i) They are negatively sloped.
- ii) Two indifference curves cannot intersect or touch.
- iii) A higher indifference curve indicates higher level of utility.
- iv) Indifference curves are strictly convex to the origin.

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The above properties of the indifference curve are based on the following Axioms of Consumer Behaviour.

★ Axioms on Consumer Behaviour

i) Completeness: Consumers' preferences are complete. The consumer compares and ranks all commodity bundles available to him. If Q_1 and Q_2 are two commodity baskets available to the consumer, then either $Q_1 > Q_2$, or $Q_2 > Q_1$, or $Q_1 \mathbf{I} Q_2$ [$>$ means preferred to, \mathbf{I} means indifferent to].

ii) Non-satiation (Dominance): A consumption basket Q_1 will be preferred to another basket Q_2 , if Q_1 contains more of at least one commodity and no less of any other commodity than Q_2 . This implies that additional utility derived from an additional unit of each commodity is always positive.

iii) Transitivity (Consistency): For any three commodity bundles Q_1 , Q_2 and Q_3 :

- a) $Q_1 > Q_2 > Q_3 \Rightarrow Q_1 > Q_3$ b) $Q_1 > Q_2 \mathbf{I} Q_3 \Rightarrow Q_1 > Q_3$
c) $Q_1 \mathbf{I} Q_2 > Q_3 \Rightarrow Q_1 > Q_3$ d) $Q_1 \mathbf{I} Q_2 \mathbf{I} Q_3 \Rightarrow Q_1 \mathbf{I} Q_3$

iv) Continuity: The consumption space is continuous, i.e, there is no gap or jump in the consumption space. Technically if Q_1 , Q_2 and Q_3 are three commodity bundles such that $Q_3 > Q_1$ and $Q_2 < Q_1$, then there should exist commodity bundle Q_i ($i=4, 5, 6, \dots$) such that $Q_i \mathbf{I} Q_1$.

v) Selection: The consumer always makes a selection of a commodity bundle, given his income and indifference map. The consumer selects the most preferred commodity bundle permissible by his income (the amount allotted for consumption expenditure).

vi) Strict Convexity: The choice set of the consumer is strictly convex. This implies that if Q_1 and Q_2 are any two commodity bundles such that $Q_1 \mathbf{I} Q_2$, i.e., $U(Q_1) = U(Q_2)$, then the utility of any linear combination of Q_1 and Q_2 is strictly greater than the utility of Q_1 (=utility of Q_2):

$$U [\lambda Q_1 + (1-\lambda) Q_2] > U (Q_1) = U(Q_2) \text{ where } 0 < \lambda < 1.$$

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vii) Divisibility and Substitutability: The commodities under consideration are infinitely divisible and substitutable. This implies that there is no gap or discontinuity in the indifference space.

viii) Reflexivity: If Q_1 is any commodity bundle, then $Q_1 \mathbf{I} Q_1$. This appears trivial at first sight but, in fact, it is not so. This axiom ensures that every commodity bundle belongs to at least one indifference set that contains the commodity bundle itself.

★ Utility Maximization

The objective of the consumer is to maximize utility or satisfaction within the limit or constraint imposed by his/her income. In this case, by income (denoted by M), we mean the total amount allotted for consumption. Let the consumer purchase only two commodities (denoted q_1 and q_2), which are substitutes to each other. The prices of these commodities are denoted by p_1 and p_2 respectively. The total amount of M is spent on these two goods. So

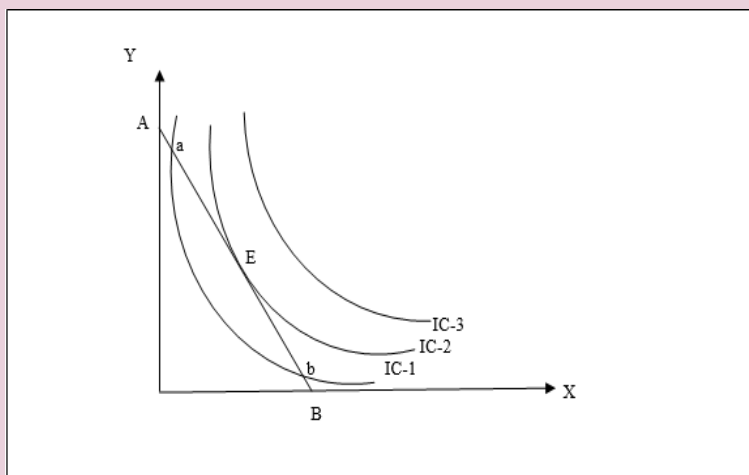
$$M = p_1q_1 + p_2q_2$$

This is the budget equation of the consumer. Let the utility function of the consumer be denoted by $U = f(q_1, q_2)$

Therefore, the consumer will

$$\text{Maximize } U = f(q_1, q_2)$$

$$\text{Subject to, } M = p_1q_1 + p_2q_2$$



Two conditions for utility maximization:

i) Necessary or first order condition:

Slope of the budget line is equal to slope of the indifference curve, i.e.

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Marginal rate of commodity substitution (MRCS) = Price ratio.

ii) Sufficient or second order condition:

The Indifference curves must be convex to the origin.

So, in the diagram E is the equilibrium point for the consumer.

★ Examples of Utility Functions

Example 1: Cobb-Douglas utility function

The function is represented as $U(x_1, x_2) = x_1^\alpha x_2^\beta$

Where α and β are positive constants that describe the preferences of the consumer. Any increasing transformation of the Cobb-Douglas utility function will represent the same preferences.

Example 2: Perfect Substitutes

For perfect substitutes the indifference curves are straight lines. For such goods, the marginal rate of substitution is a constant. In this case the consumer's preferences are represented by the utility function

$$U(x_1, x_2) = ax_1 + bx_2$$

Where a and b are positive constants. This is the general form of utility function when goods 1 and 2 are perfect substitutes.

Example 3: Perfect Complements

Suppose goods 1 and 2 are perfect complements and they are purchased in 1:1 ratio. In this case the bundles (1,1), (2,1) and (1,2) are indifferent. Therefore, the number of complete bundles of goods we have the minimum of the number of units of good 1 and the number of units of good 2. Thus, the utility function for perfect complements, when the goods are bought in 1:1 ratio, is

$$U(x_1, x_2) = \min\{x_1, x_2\}$$

In general, when goods 1 and 2 are perfect complements, the utility function is of the form

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$$U(x_1, x_2) = \min\{ax_1, bx_2\}$$

Where the positive numbers a and b denote the proportion in which the goods are consumed.

★ Irregular Indifference Curves

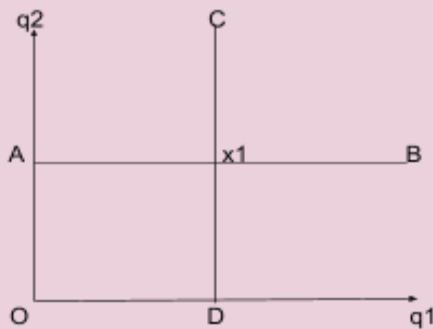
The well-behaving negatively sloped indifference curves are based on standard axioms of consumer behaviour, which, in fact, describe the choice pattern of an ideal consumer. The real consumer, however, may not conform to ideal behaviour pattern. Consumers, in fact, have idiosyncrasies. So, the behaviour pattern of real consumers may differ considerably from the ideal pattern specified by standard axioms. So, indifference curves, in reality, are likely to assume shapes, different from the shapes of ideal well-behaving ones. Some of this cases of irregularly shaped curves are discussed below.

1. Lexicographic ordering

The choice pattern of the consumer under 'Lexicographic Ordering' may be stated in the following manner.

Let the consumer purchase two commodity bundles, q_1 and q_2 . According to Lexicographic ordering, between these two bundles the bundle containing more of q_2 is preferred to one containing less of q_2 , whatever be the amount of q_2 , one with more of q_1 is preferred. This type of ordering follows the rules of arrangement of

words in Lexicons or magnitude of integers. The diagram has been depicted on the basis of the following Lexicographic Ordering: i) A bundle containing more of q_2 is preferred to one containing less of q_2 , whatever be the amount of q_1 in both the bundles.



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ii) Among bundles containing the same amount of q_2 , one with more of q_1 is preferred to those containing less of q_1 .

Now let us consider the bundle represented by the point x_1 in the above diagram.

Suppose we are to draw an indifference curve through x_1 . Now,

- a) All points lying above AB, the horizontal line passing through x_1 , are preferred to x_1 as all of them contain more of q_2 than in x_1 .
- b) x_1 is preferred to all points lying below AB as all of them contain less of q_2 than x_1 ;
- c) All points on AB, lying to the right of x_1 , is preferred to x_1 as they contain more of q_1 than x_1 although they contain the same amount of q_2 as x_1 .
- d) x_1 is preferred to all points on AB lying to the left of x_1 as they contain the same amount of q_2 but less of q_1 than x_1 .

So all points other than x_1 in the utility space is either preferred to or preferred by x_1 and, therefore, the indifference curve through x_1 contains only a single point, namely x_1 itself. This is true for all other points in the utility space under Lexicographic Ordering. Thus it is seen that we cannot draw the usual indifference curve if the preference pattern of the consumer follows Lexicographic Ordering, which violates the following Axioms of choice, namely, continuity and continuous substitution.

2. The consumer does not like simultaneous consumption of two commodities

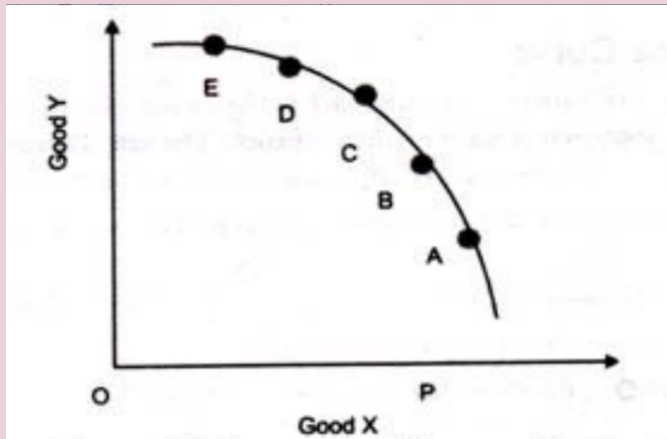
Case I. Weak Dislikes for simultaneous consumption:

The consumer likes both tea and sweets but does not like to take them simultaneously. In this case, if consumption of sweets is increased and that of tea reduced, utility from tea is reduced, as the consumer dislikes taking sweets, while taking tea (more sweets make tea less tasteful to him). On the other hand, utility from sweets is increased (the consumer dislikes taking tea while taking sweets, and so sweets become more tasteful as less tea is consumed). So, the marginal rate of substitution of tea for sweets increases as more sweets and less tea are consumed. On the other hand, $MRS_{\text{sweets, tea}}$ increases as more tea and less sweets are

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consumed. So the indifference curve for this case are concave to the origin as depicted in the following diagram.



Case II. Strong Dislike

The consumer likes both Tea and Ice Cream but never takes them together. In this case the two commodities Tea and Ice Cream can by no means be consumed together. So, the indifference curves cannot be drawn. Here, isolated points on the Tea and Ice Cream axes indicate various utility levels of the consumer. A higher point on the vertical axis, as compared to a lower point, or a rightward point on the horizontal axis, as compared to a leftward point, indicates higher level of utility. But a point on one axis cannot be compared with that on another axis unless additional information regarding the consumer's relative liking between "Tea only" and "Ice Cream only" is available. In this case the axiom of convexity, substitutability and completeness have been violated.

3. Violation of the axiom of non-satiation

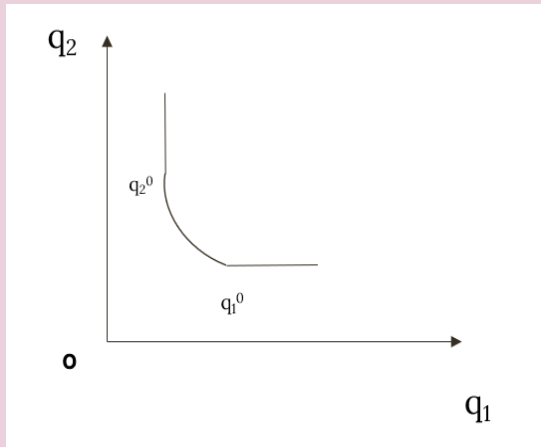
The axiom of non-satiation states that the additional consumption of a particular commodity, (consumption of other commodities remaining unchanged), adds to utility ad infinitum. This assumption appears to be a bit unrealistic. For example, continuous consumption of any food article (without any break) ultimately reduces its utility to zero or may even generate negative utility (discomfort of overeating). So let us skip the axiom of non-satiation for the time being and try to construct

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indifference curves on the basis of Marshallian Law of Diminishing Marginal Utility (without the assumption of cardinal utility).

Case I. MU of both the commodities fall to zero but does not become negative: The indifference curves, in this case are drawn below.



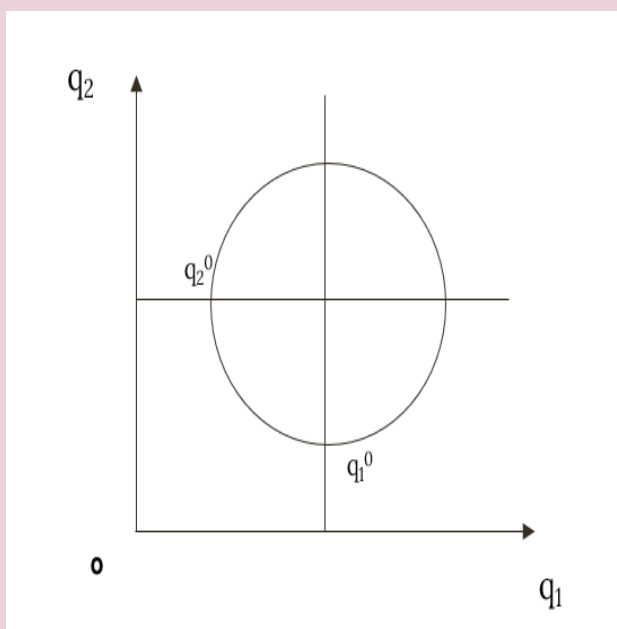
In the diagram, MU of q_1 falls to zero at the consumption level of q_1^0 and remains so beyond this level. The similar level of consumption q_2 is q_2^0 . It is not necessary that for all the indifference curves, the consumption level for satiation should be the same (here we have assumed so, simply for sake of analytical convenience). In this case

the indifference curves have segments parallel to the two axes.

Case II. MU becomes negative for both the commodities

In this case, at a certain level of consumption MU of each commodity becomes zero and beyond this level, MU of each commodity becomes negative.

In popular terms, each commodity becomes 'bad' after a certain level of consumption. In this case the indifference curves assume Circular Shapes as depicted below.



A circular indifference curve is depicted in the diagram besides. q_1 is 'good' up to consumption level q_1^0 . At this point, MU of $q_1 = 0$, and beyond this point q_1 is 'bad', i.e., MU of $q_1 < 0$.

Similarly, MU of $q_2 = 0$ at q_2^0 level of consumption of q_2 and beyond this level MU of $q_2 < 0$ (i.e., q_2 is 'bad'). The

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indifference curve depicted here has four distinct zones.

Zone 1 : In this zone (ab) the indifference curves are negatively sloped and convex to the origin, i.e., the usual shape. So, both the commodities are 'good's.

Zone 2: In this zone (bc), indifference curves slope upwards and are convex downwards in this zone. Here q_2 is 'good' and q_1 is 'bad'.

Zone 3: In this zone (ad), the indifference curves slopes upwards and are concave downwards in this region. Here q_1 is 'good' and q_2 is 'bad'.

Zone 4: in this zone (cd), each of q_1 and q_2 is 'bad'. Indifference curves are negatively sloped and concave to the origin in this zone.

So, taking all the zones, an indifference curve becomes circular. An inner circle, which is smaller than the outer one, indicates a higher level of utility. In this way, higher and higher indifference curves give us smaller and smaller circular areas, and ultimately they are reduced to a single point ('B' in the above diagram), called the 'Bliss' (which means supreme pleasure). The point 'B' has also affinity with the "Bull's Eye" in target shooting.

If the income of the consumer were such that the 'Bliss' point remains outside the budget line, we would get the usual equilibrium of the consumer by the tangency of the budget line with the highest possible indifference curve. If the budget line passes through the 'Bliss' point, equilibrium would be at the 'Bliss' point. In both the cases, the consumer will spend all his income. If, however, the budget line goes above the 'Bliss' point, the 'Bliss' point, giving the highest level of utility would be the equilibrium, and a part of the income of the consumer will remain unspent, violating one of our initial assumptions of consumer behaviour.

Indifference curves between 'Good' and 'Bad'

A commodity, which provides positive utility, may be termed as 'good' and that generating negative utility, 'bad'. For example, 'return' from some business venture is 'good', but the risk associated with the venture is 'bad'. If risk is higher, higher level of return would be required to compensate for the loss in utility due to risk, if the consumer wants to remain on the same level of satisfaction. So, in this case,

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indifference curves are upward sloping. The lowest point of each indifference curve starts from the 'return' ('good') axis. Now, these indifference curves may be of three different shapes:

- i) If the consumer becomes more intolerant to 'risk' as risk increases, marginal rate of substitution of return ('good') for risk ('bad'), would be increasing. So the indifference curve would be concave to the return ('good') axis as in diagram-1.
- ii) If we assume that with increasing 'risk', the consumer becomes more tolerant to risk, marginal rate of substitution would be falling so that the indifference curves would be convex to the 'return' axis as in diagram-2.
- iii) In case the attitude of the consumer towards risk and return is constant at all levels, indifference curves are straight lines as in diagram-3.

Concave Indifference Curve and Corner solutions

If the indifference curves are concave to the origin so that the second order condition for utility maximization is violated, the point of tangency between the budget line and an indifference curve gives a minimum level of satisfaction instead of maximum. In this situation equilibrium of the consumer is a corner solution where he purchases only one commodity as shown in the following diagram.