5.3 The Three-Level Laser System

In the last section we saw that one cannot create a steady-state population inversion between two levels just by using pumping between these levels. Thus in order to produce a steady-state population inversion, one makes use of either a three-level or a four-level system. In this section we shall discuss a three-level system.

We consider a three-level system consisting of energy levels E_1, E_2, and E_3 all of which are assumed to be nondegenerate. Let N_1, N_2, and N_3 represent the population densities of the three levels [see (Fig. 5.2)]. The pump is assumed to lift atoms from level 1 to level 3 from which they decay rapidly to level 2 through some nonradiative process. Thus the pump effectively transfers atoms from the ground level 1 to the excited level 2 which is now the upper laser level; the lower laser level being the ground state 1. If the relaxation from level 3 to level 2 is very fast, then the atoms will relax down to level 2 rather than to level 1. Since the upper level 3 is not a laser level, it can be a broad level (or a group of broad levels) so that a broadband light source may be efficiently used as a pump source (see, e.g., the ruby laser discussed in Chapter 11).

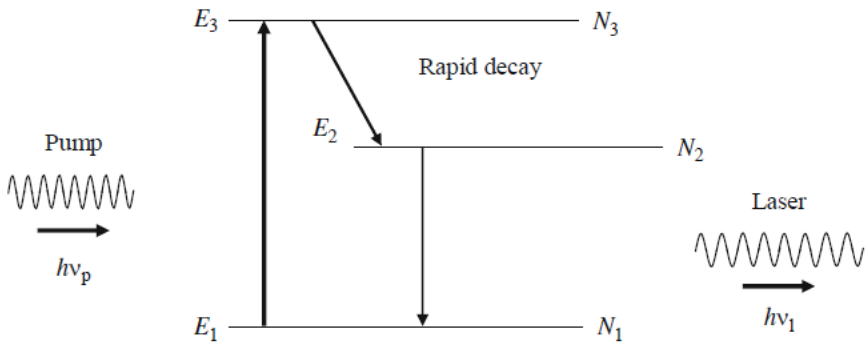

Fig. 5.2 A three-level system. The pump excites the atoms from level E_1 to level E_3 from where the atoms undergo a fast decay to level E_2. The laser action takes place between levels E_2 and E_1

If we assume that transitions take place only between these three levels then we may write

$$N = N_1 + N_2 + N_3 \tag{5.17}$$

where N represents the total number of atoms per unit volume.

We may now write the rate equations describing the rate of change of N_1, N_2 and N_3. For example, the rate of change of N_3 may be written as

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - T_{32}N_3 \tag{5.18}$$

where W_p is the rate of pumping per atom from level 1 to level 3 which depends on the pump intensity. The first term in Eq. (5.18) represents stimulated transitions between levels 1 and 3 and $T_{32}N_3$ represents the spontaneous transition from level 3 to level 2:

$$T_{32} = A_{32} + S_{32} \tag{5.19}$$

A_{32} and S_{32} correspond, respectively, to the radiative and nonradiative transition rates between levels 3 and 2. In writing Eq. (5.18) we have neglected $T_{31}N_3$ which corresponds to spontaneous transitions between levels 3 and 1 since most atoms raised to level 3 are assumed to make transitions to level 2 rather than to level 1.

In a similar manner, we may write

$$\frac{dN_2}{dt} = W_1(N_1 - N_2) + N_3T_{32} - N_2T_{21} \tag{5.20}$$

and

$$\frac{dN_1}{dt} = W_p(N_3 - N_1) + W_1(N_2 - N_1) + N_2T_{21} \tag{5.21}$$

where

$$W_1 = \frac{\pi^2 c^2}{\hbar\omega^3 n_0^2} A_{21} g(\omega) I_1 \tag{5.22}$$

represents the stimulated transition rate per atom between levels 1 and 2, I_1 is the intensity of the radiation in the $2 \to 1$ transition and $g(\omega)$ represents the lineshape function describing the transitions between levels 1 and 2. Further,

$$T_{21} = A_{21} + S_{21} \tag{5.23}$$

with A_{21} and S_{21} representing the radiative and nonradiative relaxation rates between levels 1 and 2. For efficient laser action since the transition must be mostly radiative, we shall assume $A_{21} >> S_{21}$.

At steady state we must have

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = 0 = \frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{\mathrm{d}N_3}{\mathrm{d}t} \tag{5.24}$$

From Eq. (5.18) we obtain

$$N_3 = \frac{W_p}{W_p + T_{32}} N_1 \tag{5.25}$$

Using Eqs. (5.20), (5.21), and (5.25) we get

$$N_2 = \frac{W_1(T_{32} + W_p) + W_p T_{32}}{(W_p + T_{32})(W_1 + T_{21})} N_1 \tag{5.26}$$

Thus from Eqs. (5.17), (5.25), and (5.26) we get

$$\frac{N_2 - N_1}{N} = \frac{[W_p(T_{32} - T_{21}) - T_{32}T_{21}]}{[3W_pW_1 + 2W_pT_{21} + 2T_{32}W_1 + T_{32}W_p + T_{32}T_{21}]} \qquad (5.27)$$

From the above equation, one may see that in order to obtain population inversion between levels 2 and 1, i.e., for $(N_2 - N_1)$ to be positive, a necessary (but not sufficient) condition is that

$$T_{32} > T_{21} \qquad (5.28)$$

Since the lifetimes of levels 3 and 2 are inversely proportional to the relaxation rates, according to Eq. (5.28), the lifetime of level 3 must be smaller than that of level 2 for attainment of population inversion between levels 1 and 2. If this condition is satisfied then according to Eq. (5.27), there is a minimum pumping rate required to achieve population inversion which is given by

$$W_{pt} > \frac{1}{(\tau_{21} - \tau_{32})}$$

{When the population inversion is achieved then the condition is that the numerator of the equation (5.27) is always positive i.e

$$[W_{pt}(T_{32} - T_{21}) - T_{32}T_{21}] > 0$$

⇨ $[W_{pt}] > \frac{T_{32}T_{21}}{(T_{32}-T_{21})}$

⇨ $[W_{pt}] > \frac{T_{21}}{\left(1-\frac{T_{21}}{T_{32}}\right)}$ 5.29

We know that T_{ij} (i,j=3,2) is inversely proportional to the life time of levels 3 and 2($\tau_{i,j}\{i,j = 3,2\}$)

So we can write the condition$\tau_{21} > \tau_{32}$. Then the equation 5.29 can be written by

$[W_{pt}] > \frac{1/\tau_{21}}{\left(1-\frac{\tau_{32}}{\tau_{21}}\right)}$

⇨ $[W_{pt}] > \frac{1}{(\tau_{21}-\tau_{32})}$

Here, we see that $\tau_{21} > \tau_{32}$,so the population inversion must be achieved between the levels 2 and 1.

The minimum pumping rate required to achieve population inversion is the threshold condition which is given by

$$[W_{pt}] = \frac{1}{(\tau_{21} - \tau_{32})}$$

5.4 The Four-Level Laser System

In the last section we found that since the lower laser was the ground level, one has to lift more than 50% of the atoms in the ground level in order to obtain population inversion. This problem can be overcome by using another level of the atomic system and having the lower laser level also as an excited level. The four-level laser

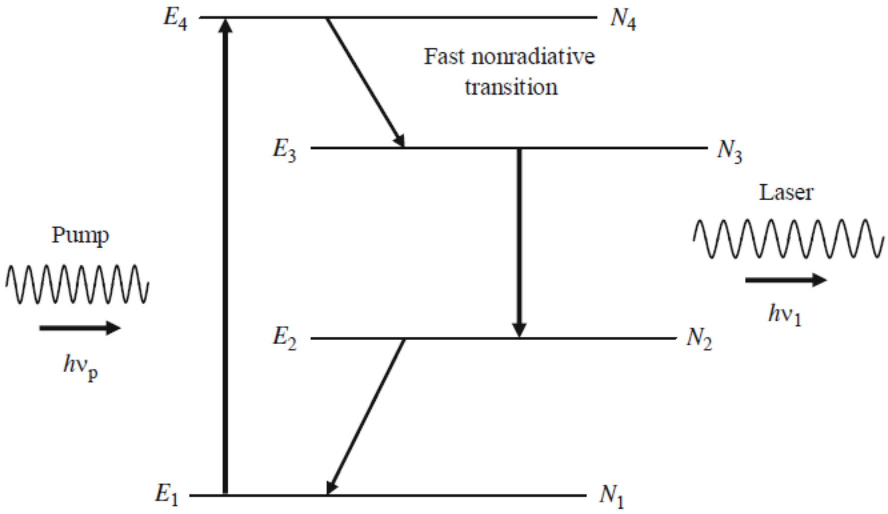

Fig. 5.3 A four-level system; the pump lifts atoms from level E_1 to level E_4 from where they decay rapidly to level E_3 and laser emission takes place between levels E_3 and E_2. Atoms drop down from level E_2 to level E_1

system is shown in Fig 5.3. Level 1 is the ground level and levels 2, 3, and 4 are excited levels of the system. Atoms from level 1 are pumped to level 4 from where they make a fast nonradiative relaxation to level 3. Level 3 which corresponds to the upper laser level is usually a metastable level having a long lifetime. The transition from level 3 to level 2 forms the laser transition. In order that atoms do not accumulate in level 2 and hence destroy the population inversion between levels 3 and 2, level 2 must have a very small lifetime so that atoms from level 2 are quickly removed to level 1 ready for pumping to level 4. If the relaxation rate of atoms from level 2 to level 1 is faster than the rate of arrival of atoms to level 2 then one can obtain population inversion between levels 3 and 2 even for very small pump powers. Level 4 can be a collection of a large number of levels or a broad level. In such a case an optical pump source emitting over a broad range of frequencies can be used to pump atoms from level 1 to level 4 effectively. In addition, level 2 is required to be sufficiently above the ground level so that, at ordinary temperatures, level 2 is almost unpopulated. The population of level 2 can also be reduced by lowering the temperature of the system.

We shall now write the rate equations corresponding to the populations of the four levels. Let N_1, N_2, N_3, and N_4 be the population densities of levels 1, 2, 3, and 4, respectively. The rate of change of N_4 can be written as

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - T_{43}N_4 \tag{5.44}$$

where, as before, W_pN_1 is the number of atoms being pumped per unit time per unit volume, W_pN_4 is the stimulated emission rate per unit volume,

$$T_{43} = A_{43} + S_{43} \tag{5.45}$$

is the relaxation rate from level 4 to level 3 and is the sum of the radiative (A_{43}) and nonradiative (S_{43}) rates. In writing Eq. (5.44) we have neglected (T_{42}) and (T_{41}) in comparison to (T_{43}), i.e., we have assumed that the atoms in level 4 relax to level 3 rather than to levels 2 and 1.

Similarly, the rate equation for level 3 may be written as

$$\frac{\mathrm{d}N_3}{\mathrm{d}t} = W_1(N_2 - N_3) + T_{43}N_4 - T_{32}N_3 \tag{5.46}$$

where

$$W_1 = \frac{\pi^2 c^2}{\hbar\omega^3 n_0^2} A_{32} g_1(\omega) I_1 \tag{5.47}$$

represents the stimulated transition rate per atom between levels 3 and 2 and the subscript 1 stands for laser transition; $g_1(\omega)$ is the lineshape function describing the $3 \leftrightarrow 2$ transition and I_1 is the intensity of the radiation at the frequency $\omega = (E_3 - E_2)/\hbar$. Also

$$T_{32} = A_{32} + S_{32} \tag{5.48}$$

is the net spontaneous relaxation rate from level 3 to level 2 and consists of the radiative (A_{32}) and the nonradiative (S_{32}) contributions. Again we have neglected any spontaneous transition from level 3 to level 1. In a similar manner, we can write

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -W_1(N_2 - N_3) + T_{32}N_3 - T_{21}N_2 \tag{5.49}$$

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = -W_\mathrm{p}(N_1 - N_4) + T_{21}N_2 \tag{5.50}$$

where

$$T_{21} = A_{21} + S_{21} \tag{5.51}$$

is the spontaneous relaxation rate from $2 \rightarrow 1$.

Under steady-state conditions

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{\mathrm{d}N_3}{\mathrm{d}t} = \frac{\mathrm{d}N_4}{\mathrm{d}t} = 0 \tag{5.52}$$

We will thus get four simultaneous equations in N_1, N_2, N_3, and N_4 and in addition we have

$$N = N_1 + N_2 + N_3 + N_4 \tag{5.53}$$

for the total number of atoms per unit volume in the system.

From Eq. (5.44) we obtain, setting $\mathrm{d}N_4/\mathrm{d}t = 0$

$$\frac{N_4}{N_1} = \frac{W_\mathrm{p}}{(W_\mathrm{p} + T_{43})} \tag{5.54}$$

If the relaxation from level 4 to level 3 is very rapid then $T_{43} \gg W_p$ and hence $N_4 \ll N_1$. Using this approximation in the remaining three equations we can obtain for the population difference,

$$\frac{N_3 - N_2}{N} \approx \frac{W_p(T_{21} - T_{32})}{W_p(T_{21} + T_{32}) + T_{32}T_{21} + W_l(2W_p + T_{21})} \tag{5.55}$$

Thus in order to be able to obtain population inversion between levels 3 and 2, we must have

$$T_{21} > T_{32} \tag{5.56}$$

i.e., the spontaneous rate of deexcitation of level 2 to level 1 must be larger than the spontaneous rate of deexcitation of level 3 to level 2.

If we now assume $T_{21} \gg T_{32}$, then from Eq. (5.55) we obtain

$$\frac{N_3 - N_2}{N} \approx \frac{W_p}{W_p + T_{32}} \frac{1}{1 + W_l(T_{21} + 2W_p)/T_{21}(W_p + T_{32})} \tag{5.57}$$

From the above equation we see that even for very small pump rates one can obtain population inversion between levels 3 and 2. This is contrary to what we found in a three-level system, where there was a minimum pump rate, W_{pt}, required to achieve inversion. The first factor in Eq. (5.57) which is independent of W_l [i.e., independent of the intensity of radiation corresponding to the laser transition – see Eq. (5.47)] – gives the small signal gain coefficient whereas the second factor in Eq. (5.57) gives the saturation behavior.