

A general theory of deduction will have two objectives:

- (1) to explain the relations between premises and conclusions in deductive arguments, and
- (2) to provide techniques for discriminating between valid and invalid deductions.

Two logical theories: Classical (or Aristotelian) logic and modern, symbolic, or mathematical logic.

The relations of classes of things are not central for modern logicians as they were for Aristotle and his followers. Instead, logicians look now to the internal structure of propositions and arguments, and to the logical links—very few in number—that are critical in all deductive argument.

An artificial symbolic language is required.

In a natural language, words may be vague or equivocal, the construction of arguments may be ambiguous, metaphors and idioms may confuse or mislead, emotional appeals may distract.

=====

### **Simple statement**

A statement that does not contain any other statement as a component.

### **Compound statement**

A statement that contains two or more statements as components.

For a part of a statement to be a **component** of that statement, two conditions must be satisfied:

- (1) The part must be a statement in its own right; *and*
- (2) if the part is replaced in the larger statement by any other statement, the result of that replacement must be meaningful—it must make sense.

Example: The man who awarded **Dr. Kalam is the president of India**.

The man who awarded **Delhi is a polluted city**.

Modern logic begins by first identifying the fundamental **logical connectives** on which deductive arguments depend. Using these connectives, a general account of such arguments is given, and methods for testing the validity of arguments are developed.

## **1. Conjunction সংযোগ (and)**

We can form the **conjunction** of two statements by placing the word “and” between them;

The two statements so combined are called **conjuncts (সংযোগী)**.

“Charlie is neat **and** Charlie is sweet,” is a conjunction.

“Lincoln **and** Grant were contemporaries,” is not a conjunction, but a simple statement expressing a relationship.

Symbol for conjunction •

“Charlie is neat • Charlie is sweet.”

**Truth Value সত্যমূল্য:** every statement has a **truth value**, where the truth value of a true statement is *true*, and the truth value of a false statement is *false*.

**Truth-functional compound statement: সত্যাপেক্ষ যৌগিক বচনঃ** A compound statement whose truth value is determined wholly by the truth values of its components.

The truth value of the conjunction of two statements is determined wholly and entirely by the truth values of its two conjuncts. If both its conjuncts are true, the conjunction is true; otherwise it is false.

For this reason, a conjunction is said to be a **truth-functional compound statement**, and its conjuncts are said to be **truth-functional components** of it.

Not every compound statement is truth-functional.

Example: “Othello believes that **Desdemona loves Cassio**”.

**Truth-functional component সত্যাপেক্ষ অবয়ব** Any component of a compound statement whose replacement there by any other statement having the same truth value would leave the truth value of the compound statement unchanged.

**Truth-functional connective • সত্যাপেক্ষ যোজক**

Given any two statements,  $p$  and  $q$ , there are only four possible sets of truth values they can have.

Where  $p$  is true and  $q$  is true, is true.

Where  $p$  is true and  $q$  is false, is false.

Where  $p$  is false and  $q$  is true, is false.

Where  $p$  is false and  $q$  is false, is false.

Truth Table:

P	Q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

A statement of the form  $p \bullet q$  is true if and only if  $p$  is true and  $q$  is true.

## 2. Negation $\sim$ (not)

M: "All humans are mortal,"

$\sim$ M: "Not all humans are mortal," "Some humans are not mortal," "It is false that all humans are mortal," and "It is not the case that all humans are mortal"

Truth table

P	$\sim p$
T	F
F	T

## 3. Disjunction বৈকল্পিক বচনঃ (or) $\vee$

The two component statements so combined are called *disjuncts* (or *alternatives*) বিকল্প।

**Weak or Inclusive disjunction দুর্বল বৈকল্পিক বচন:** A compound statement asserting inclusive disjunction is true if one or the other or both disjuncts are true; Normally called simply "disjunction," it is also called "weak disjunction" and is symbolized by the wedge,  $\vee$

অথবা  $\rightarrow$  অন্তত এক

"Fees will not be taken in the event of sickness or unemployment."

**Strong or Exclusive disjunction: সবল বা বিসংবাদী বা পরস্পর বর্জনকারী বিকল্পঃ**

অথবা  $\rightarrow$  অন্তত এক কিন্তু উভয় নয়

রাম জীবিত অথবা মৃত

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

## Conditional Statements প্রাকল্পিক বচন :

If.....then

যদি রাম আসে তাহলে অনিতাও আসবে।

(পূর্বগ)

(অনুগ)

If Ram comes, **then** Anita will also come.

Antecedent

Consequent

A conditional statement asserts that in any case in which its antecedent is true, its consequent is also true.

It does not assert that its antecedent is true, but only that if its antecedent is true, then its consequent is also true.

It does not assert that its consequent is true, but only that its consequent is true if its antecedent is true.

The essential meaning of a conditional statement is the relationship asserted to hold between the antecedent and the consequent, in that order.

A conditional statement asserts some implication (প্রতিপত্তি).

Consider the following four conditional statements, each of which seems to assert a different type of implication, and to each of which corresponds a different sense of "if-then":

- A.** If all humans are mortal and Socrates is a human, then Socrates is mortal.
- B.** If Leslie is a bachelor, then Leslie is unmarried.
- C.** If this piece of blue litmus paper is placed in acid, then this piece of blue litmus paper will turn red.
- D.** If State loses the homecoming game, then I'll eat my hat.

Any conditional statement, "If  $p$  then  $q$ ," is known to be false only if its antecedent is true and its consequent false.

**Material implication** A truth-functional relation (symbolized by the horseshoe,  $\supset$ ) that may connect two statements. The statement " $p$  materially implies  $q$ " is true when either  $p$  is false, or  $q$  is true.

For any conditional, "If  $p$  then  $q$ ," to be true, the statement which is the negation of the conjunction  $\sim (p \cdot \sim q)$  of its antecedent with the negation of its consequent, must also be true.

Material implication constitutes a fifth type that may be asserted in ordinary discourse.

Consider the remark, "If Hitler was a military genius, then I'm a monkey's uncle."

It is quite clear that it does not assert logical, definitional, or causal implication.

It cannot represent a decisional implication, because it scarcely lies in the speaker's power to make the consequent true.

No "real connection," whether logical, definitional, or causal, obtains between antecedent and consequent here.

The full meaning of the present conditional seems to be the denial that "Hitler was a military genius" is true when "I'm a monkey's uncle" is false. Because the latter is so obviously false, the conditional must be understood to deny the former.

<b>p</b>	<b>q</b>	<b><math>p \supset q</math></b>
T	T	T
T	F	F
F	T	T
F	F	T

If he has a good lawyer, then he will be acquitted.

- 1) If he has a good lawyer, he will be acquitted.
- 2) He will be acquitted if he has a good lawyer.
- 3) There is no way he won't be acquitted if he has a good lawyer.

Any of these is symbolized as  $L \supset A$ .

Any conditional statement, "If  $p$  then  $q$ ," is known to be false if the conjunction  $p \cdot \sim q$  is known to be true. For a conditional to be true, then, the indicated conjunction must be false; that is, its negation  $\sim (p \cdot \sim q)$  must be true.

<b>p</b>	<b>q</b>	<b><math>\sim q</math></b>	<b><math>p \cdot \sim q</math></b>	<b><math>\sim (p \cdot \sim q)</math></b>	<b><math>p \supset q</math></b>
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

**Material equivalence** is the truth-functional connective that asserts that the statements it connects have the *same* truth value.

Two statements that are equivalent in truth value, therefore, are materially equivalent.

One straightforward definition is this: Two statements are *materially equivalent* when they are both true, or both false.

Any two statements that are materially equivalent must imply one another, because they are either both true or both false. দ্বিপাক্ষিক

<b>p</b>	<b>q</b>	<b><math>p \equiv q</math></b>
T	T	T
T	F	F
F	T	F
F	F	T