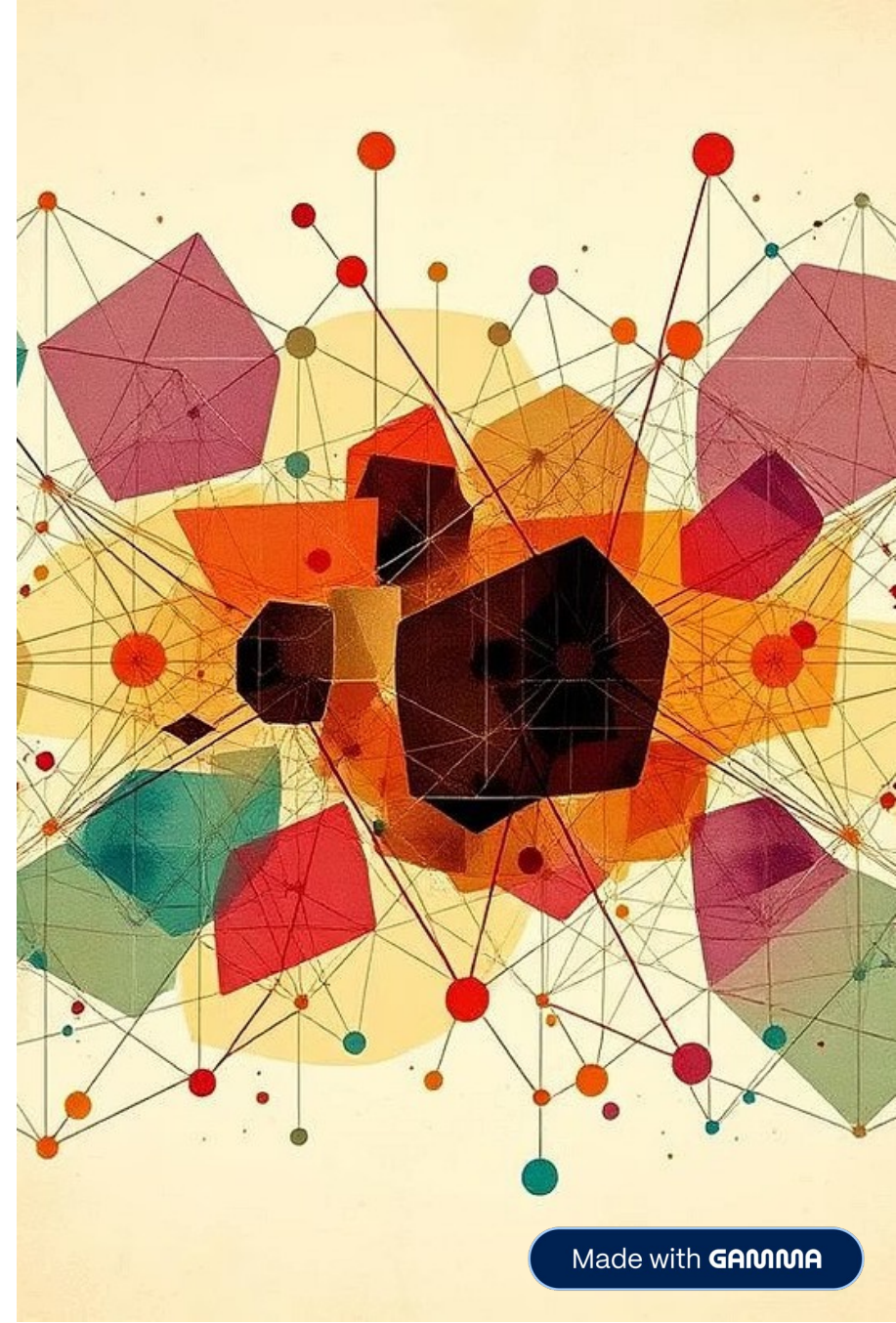


Introduction to Set Theory

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What is a Set and Different Types of Sets

Definition

An unordered collection of distinct objects (elements). Notation: $a \in A$ (element belongs to set).



Finite Set

Fixed element count (e.g., $A = \{1, 3, 5\}$; $|A| = 3$).



Infinite Set

Uncountable elements (e.g., natural numbers \mathbb{N}).



Empty Set (\emptyset)

No elements; $|\emptyset| = 0$.



Singleton Set

One element (e.g., $\{7\}$).



Equal Sets

Identical elements (e.g., $\{1, 3\} = \{3, 1\}$).

Basic Set Operations

Union ($A \cup B$)

Elements in A, B, or both. Ex: $\{2, 3\} \cup \{3, 4\} = \{2, 3, 4\}$.

Intersection ($A \cap B$)

Elements common to A and B. Ex: $\{2, 3\} \cap \{3, 4\} = \{3\}$.

Set Difference ($A \setminus B$)

Elements in A but not in B. Ex: $\{2, 3\} \setminus \{3, 4\} = \{2\}$.

Complement (A^c)

Elements in U but not in A ($A^c = U \setminus A$).

i Disjoint Sets: No common elements ($A \cap B = \emptyset$). Example: Even and odd integers.

Properties of Set Operations

1 Commutative

$$A \cup B = B \cup A, A \cap B = B \cap A.$$

3 Distributive

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

2 Associative

$$(A \cup B) \cup C = A \cup (B \cup C), (A \cap B) \cap C = A \cap (B \cap C).$$

4 Identity/Domination

$$A \cup \emptyset = A, A \cap U = A, A \cup U = U, A \cap \emptyset = \emptyset.$$

Cartesian Product and Its Properties

Definition

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

(ordered pairs).

Example:

$$\{1, 2\} \times \{p, q\} = \{(1, p), (1, q), (2, p), (2, q)\}$$

Properties

- Cardinality: $|A \times B| = |A| \times |B|$.
- Non-commutative:
 $A \times B \neq B \times A$ (unless $A = B$).
- Associative:
 $A \times (B \times C) = (A \times B) \times C$

Applications: Defines relations (e.g., coordinate systems).



| | | | |
|-------|----|--------|---------|
| | | 6 | = 0 |
| | | + 0 | + 3 |
| | 99 | + 1 | = 1 + 0 |
| J + | ▷= | 11, | + = 0 |
| 2.2 + | S= | 16 - x | + 10 |
| 2.1 > | S= | + 2 + | + 1 + |
| 3.2 > | 6= | + 2 + | + 1)8 |
| 4.2 > | S= | 4, 12 | + 2 + |
| 4.5 > | 0= | 121 | + 6 + |
| 4.5 > | S= | + 5 + | + 2 + |
| 2.2 + | ▷= | 1 10 | + 20 |

Cartesian Product of Sets

So far, we have considered some operations on sets; operations like Union of sets, intersection of set, compliment of a set, difference of two sets and symmetric difference of two sets but nothing has been said about product of two sets.

The product possible with sets is what we called Cartesian Product (\times). In this Module, we will be looking at the Cartesian product of sets.



| | | | |
|--------|----|--------|---------|
| | | 9 | |
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| 3, 2 > | 6= | + 2 + | + 1) 8 |
| 4.2 > | S= | 4, 12 | + 2 + |
| 4.5 > | O= | 121 | + 6 + |
| 4, 5 > | S= | + 5 + | + 2 + |
| 2.2 + | >= | 1 10 | + 20 |

Example

Let **A** and **B** be any 2 finite sets, the Cartesian product of set **A** and set **B** denoted as $A \times B$ is defined as the set of ordered pair (a, b) such that $a \in A$ and $b \in B$. i.e.

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

e.g.

Let $A = a, b, c, d$ and $B = 1, 2, 3$, then

$$A \times B = \left\{ \begin{array}{l} (a, 1), (a, 2), (a, 3) \\ (b, 1), (b, 2), (b, 3) \\ (c, 1), (c, 2), (c, 3) \\ (d, 1), (d, 2), (d, 3) \end{array} \right\}$$



| | | | |
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Also,

$$B \times A = \left\{ \begin{array}{l} (1, a), (1, b), (1, c), (1, d) \\ (2, a), (2, b), (2, c), (2, d) \\ (3, a), (3, b), (3, c), (3, d) \end{array} \right\}$$

Clearly, it is seen that $A \times B \neq B \times A$ which is to say that the operation Cartesian product is not commutative.

It could only be commutative if $A = B$ or A or B is empty (ϕ)



| | | | |
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It could also be established that Cartesian product is not associative, i.e. for any 3 finite sets **A**, **B** and **C**

$$A \times (B \times C) \neq (A \times B) \times C$$

Cartesian product satisfies the following with respect to intersection $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

The above is not true in most cases if we replace Intersection (\cap) with Union (\cup).



| | | | |
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◆ Key Properties

- Not commutative:

$$A \times B \neq B \times A$$

- Number of elements:

If $|A| = m$ and $|B| = n$, then

$$|A \times B| = m \times n$$

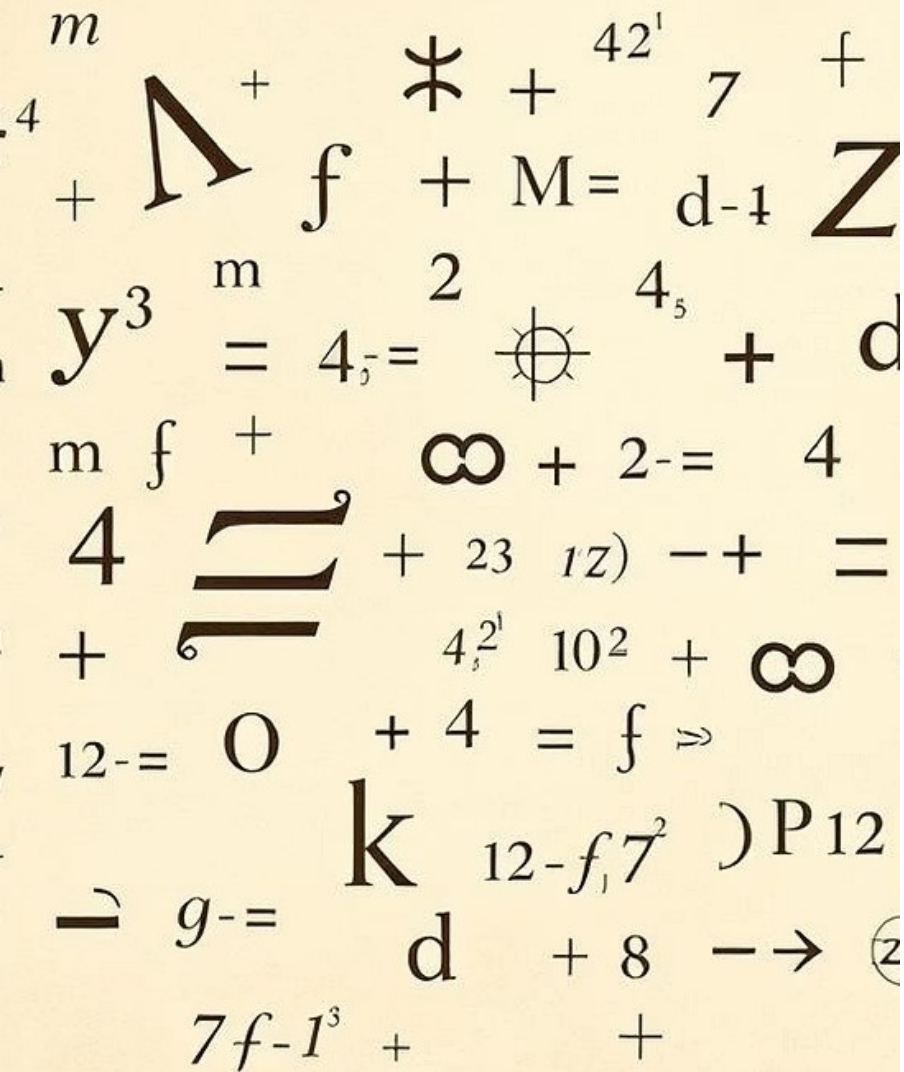
- Empty set:

If either A or B is empty, then

$$A \times B = \emptyset$$



| | | | |
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Open and Closed Sets

Open Set

Every point has a neighborhood entirely within the set. Example:
 $(0, 1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$ (open interval).

Key Notes: \mathbb{R}^n and \emptyset are both open and closed. Critical for continuity and optimization in economics.


Closed Set

Contains all its limit points. Example:
 $[0, 1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ (closed interval).

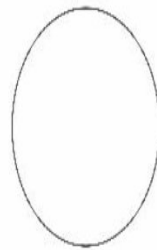
Open Set


✓ Visual Idea

Open Set




Closed Set






open set



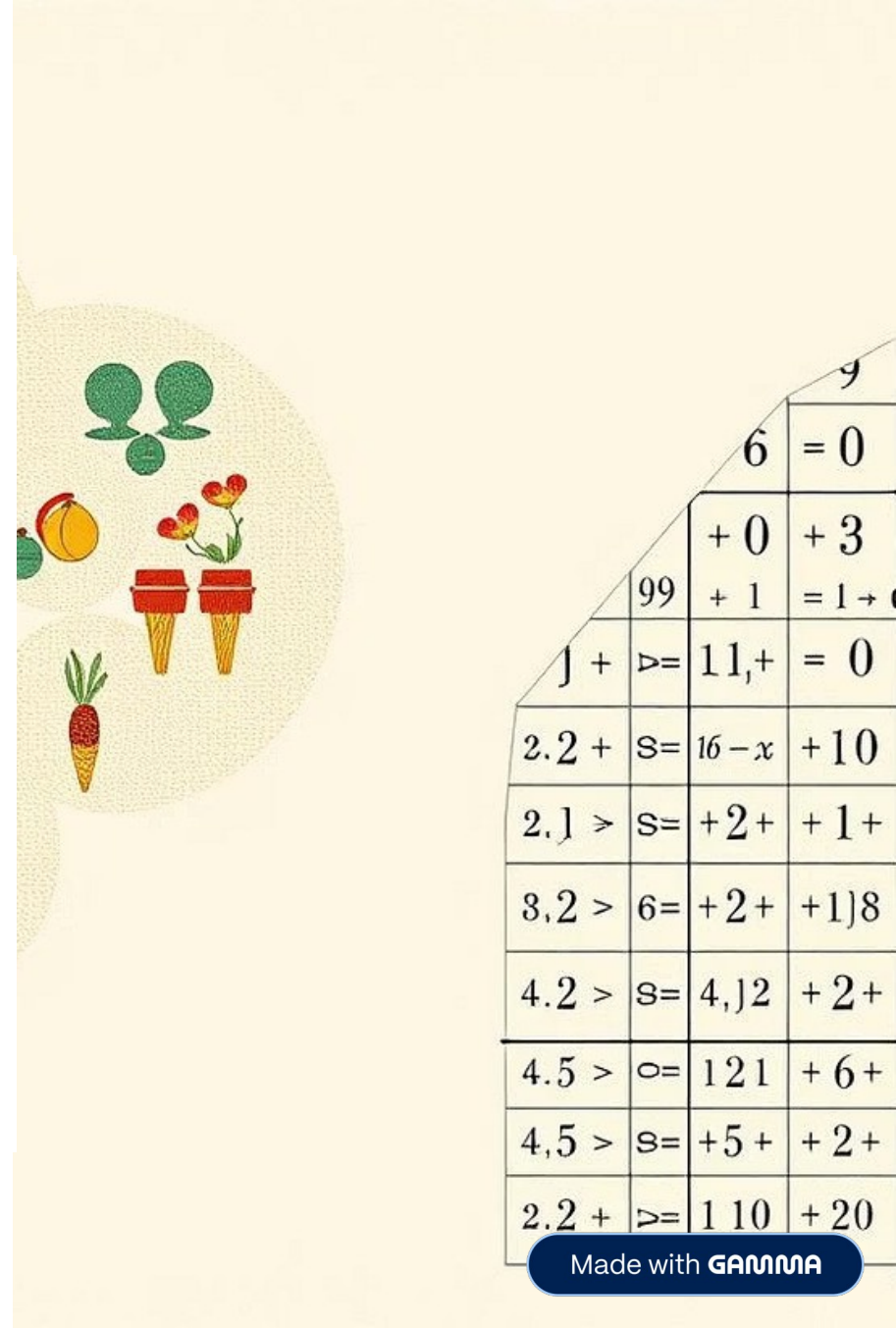
closed set



general set

| | |
|---|---|
| <p>Open Interval</p> <p>$a < x < b$ (a, b)</p> <p>Closed Interval</p> <p>$a \leq x \leq b$ $[a, b]$</p> | <p>Infinite Interval</p> <p>$x < a$</p> <p>$x \leq a$</p> <p>$x > b$</p> <p>$x \geq b$</p> |
| <p>Intervals open at one end and closed at other end</p> <p>$a < x \leq b$ $(a, b]$</p> <p>$a \leq x < b$ $[a, b)$</p> | |

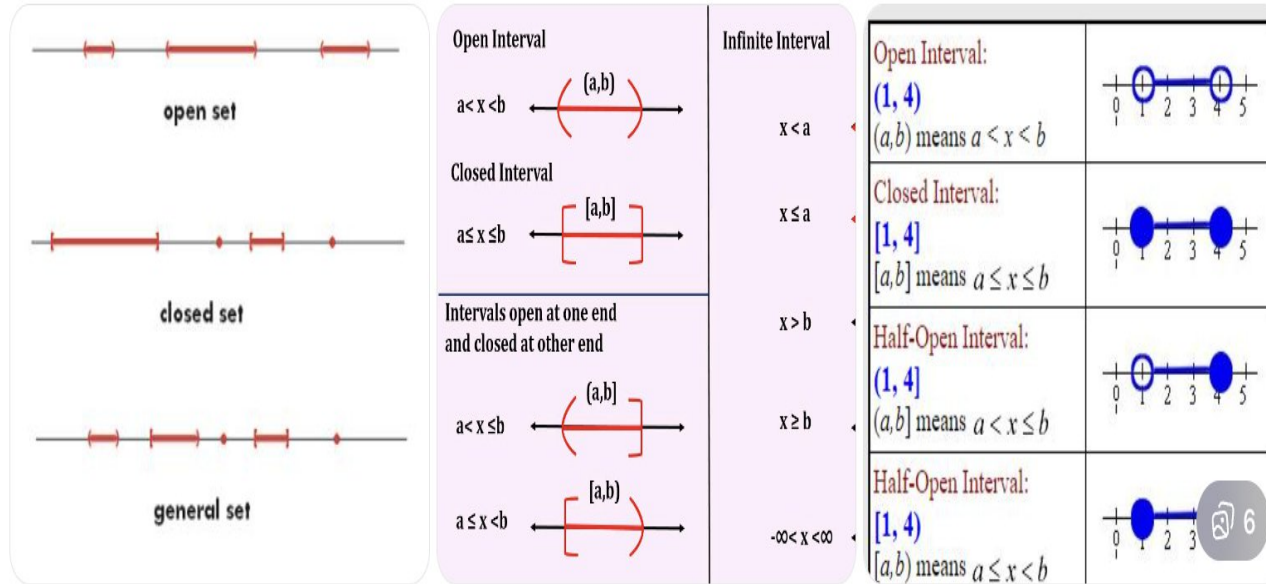
👉 Endpoints are shown as hollow (not included)



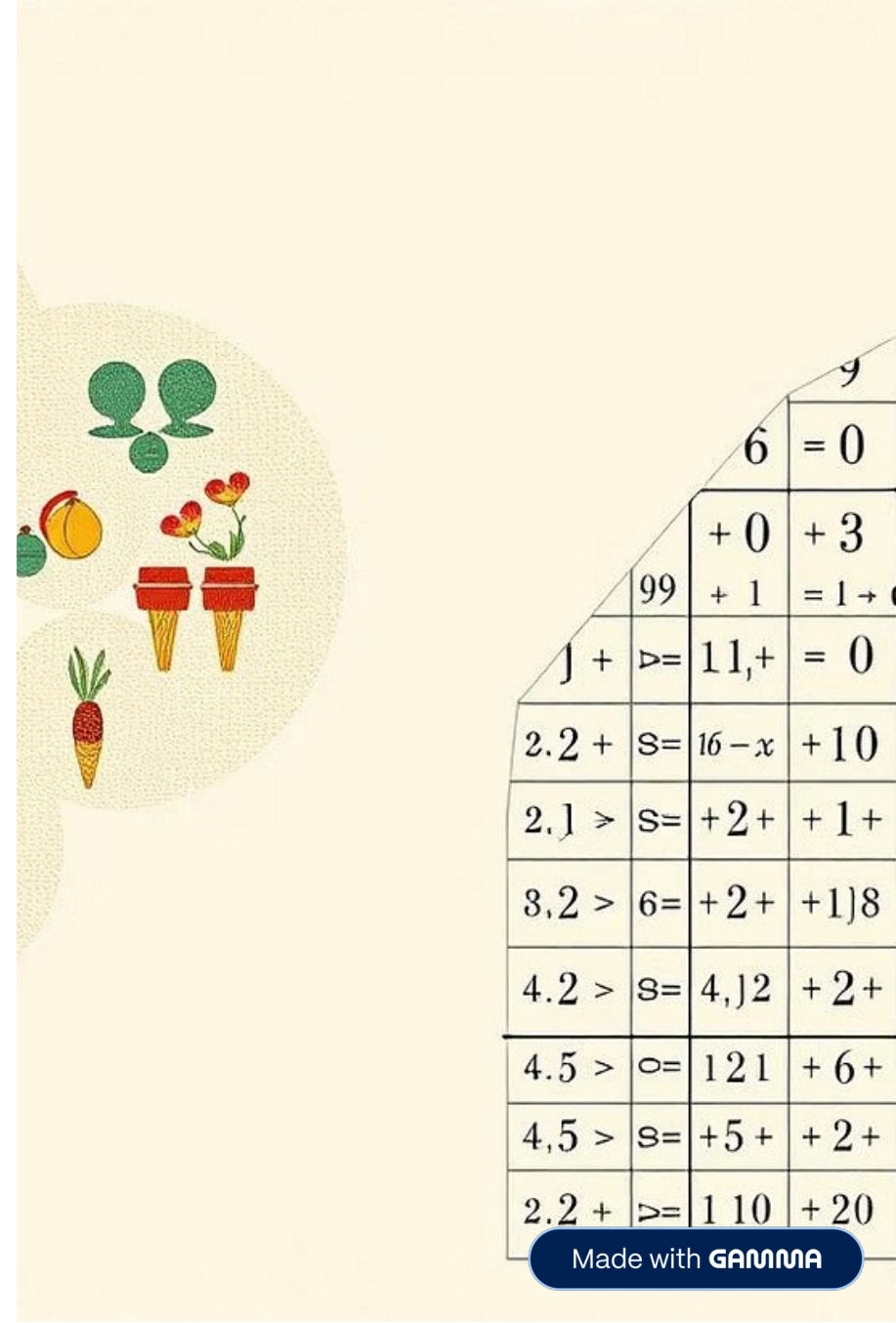
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Closed Set

✓ Visual Idea



☞ Endpoints are shown as filled (included)



◆ Key Differences

| Basis | Open Set | Closed Set |
|-----------------|------------------|----------------------|
| Boundary points | Not included | Included |
| Neighbourhood | Fully inside set | May include boundary |
| Example | $(0,1)$ | $[0,1]$ |

| | | |
|--------------|---------|---------|
| $4.2 > 8 =$ | $4, 12$ | $+ 2 +$ |
| $4.5 > 0 =$ | 121 | $+ 6 +$ |
| $4.5 > 8 =$ | $+ 5 +$ | $+ 2 +$ |
| $2.2 + \geq$ | $1 10$ | $+ 20$ |

Convex Sets

Definition

For any $x, y \in S$ and $\alpha \in [0, 1]$,

$\alpha x + (1 - \alpha)y \in S$.

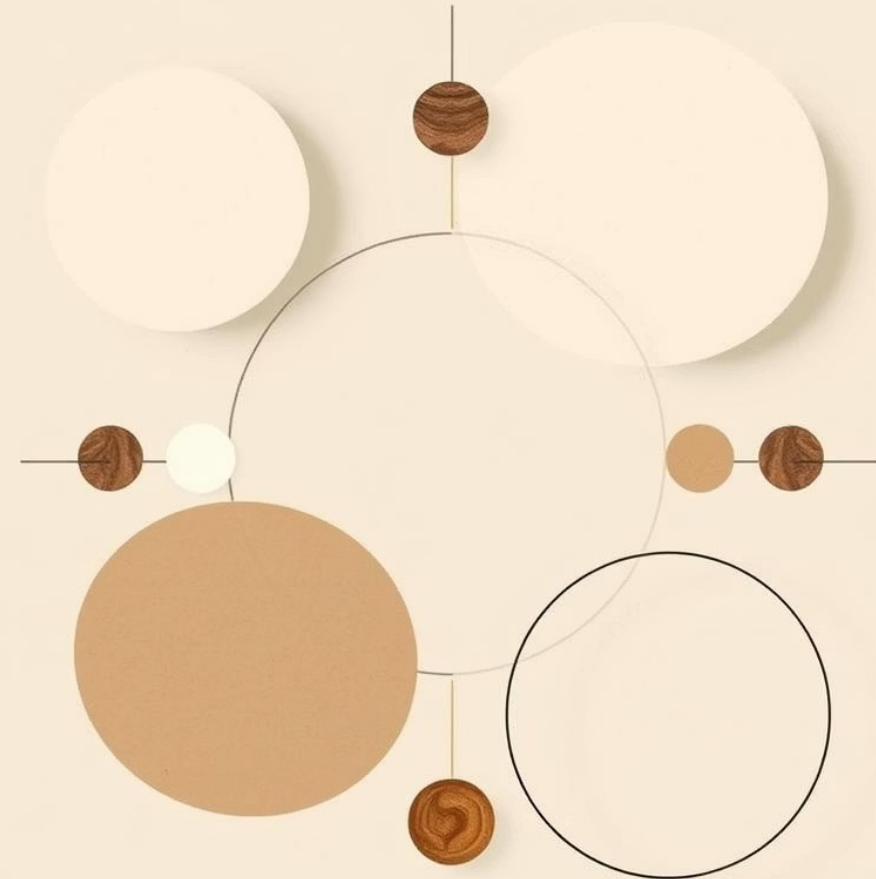
Geometric interpretation: Line segment between points lies in the set.

Economic Application: Ensures "well-behaved" optimization (e.g., no local maxima in utility functions).

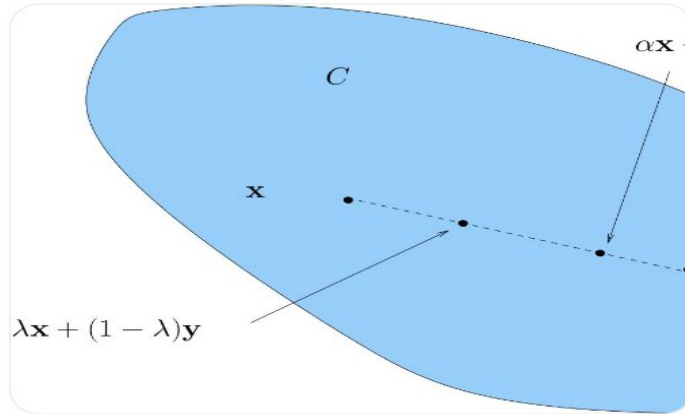
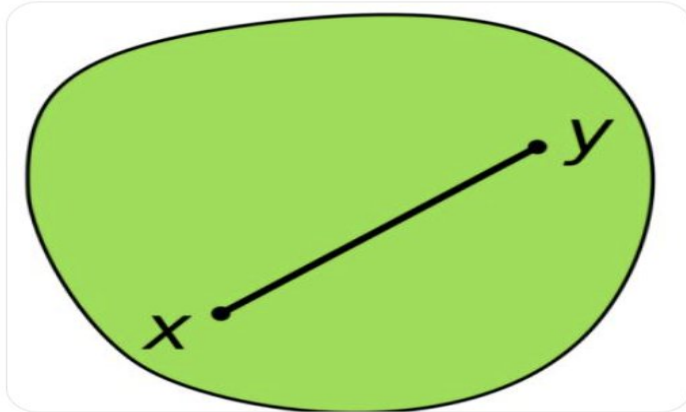
Examples

Convex: Circle (with interior), line segment.

Non-convex: Star-shaped set, crescent.



✓ Convex Sets



Convex Sets

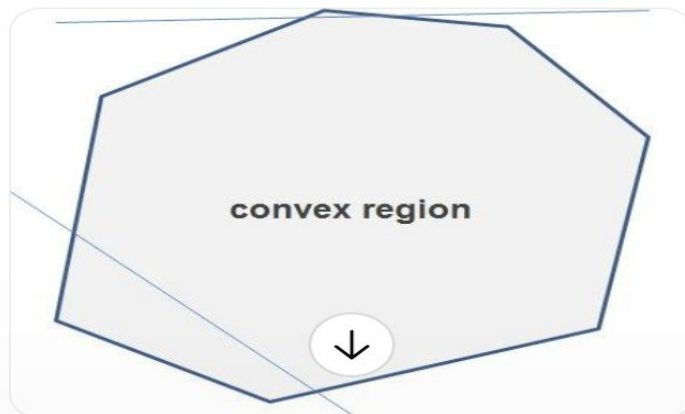
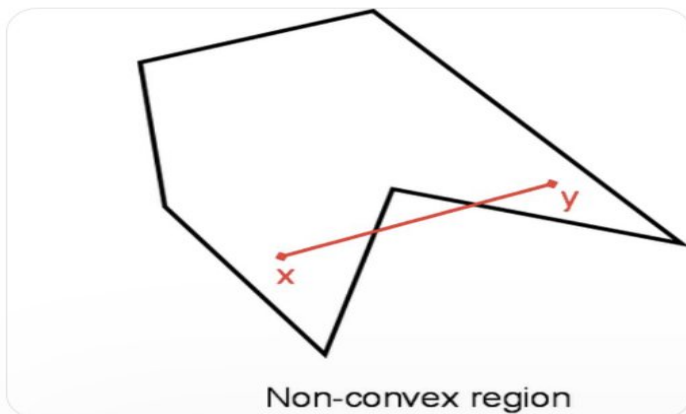
Definition #3: A set C is convex if a line segment between any two points in C lies in C .

Ex: Which of the below are convex sets?

The set on the left is convex. The set on the right is not.

- A **solid circle (disc)**
- A **square or rectangle**
- Any **interval** on the real line

✗ Non-Convex Sets



Convex Set



Thank You!

We appreciate your attention and hope this presentation provided valuable insights into Set Theory and Matrices.